# Evolution of Degree of Polarization in Presence of Polarization Mode dispersion in Single Mode Fibers

Hassan Abid Yasser(1) Nizar Salim Shnan(2)

(1) Thi-Qar University/ College of Science
(2) Babylon University/College of Science for Women

**Abstract**

In the presence of polarization mode dispersion (PMD) in single mode fiber, the degree of polarization (DOP) are affected randomly depending on the amount of PMD and the initial pulse width. In this paper, we are derived a novel analytical expression of the DOP that may be used to expect the reconstructed polarization for a single section. Thereafter, this expression was generalized for any number of concatenation sections in order to cover the randomness of the local variations of the direction and value of the PMD vector.

**Keywords**: PMD, DOP, SOP



## 1. Introduction

Polarization mode dispersion (PMD) is caused by optical birefringence and is a fundamental property of single-mode optical fiber and fiber-optic components in which signal energy at a wavelength is resolved into two orthogonal polarization modes of slightly different propagation velocities [1].

PMD results in pulse broadening and distortion thereby leading to system performance degradation. Unlike the chromatic dispersion, PMD varies stochastically in time making it particularly difficult to assess, counter or cope with [2,3]. Active research is being conducted by different groups on different issues of PMD for more than a decade. The objective of the PMD research is to understand the stochastic nature of PMD thoroughly through analytical analysis, simulations and/or analysis of measured data and determine an efficient means for mitigating PMD effects on long-haul fiber networks. To ensure signal quality on their fiber at higher rates, network engineers must anticipate the impact of PMD on various fiber routes [4,5].

Design of a reliable network requires a good model of the PMD characteristics on each link. An understanding of the temporal and spectral variability of both the differential group delay (DGD) and principal states of polarization (PSPs) is required to specify appropriate transmission parameters and also the required speed of PMD compensators [1,3]. Factors such as the mean DGD, PMD correlation time and bandwidth, as well as second-order effects together with performance prediction models can provide this understanding. Also, a solid understanding of PMD-induced system outages will help engineers and researchers to develop new and cost-efficient mitigation alternatives to PMD compensators [4]. Another recently developed PMD monitoring technique is based on the degree of polarization (DOP) of the received optical signal. PMD can depolarize the optical signal [5,6]. This in turn reduces the DOP. The merits of using DOP evaluation in the PMD monitoring mechanism are several in number. DOP is bit-rate independent and largely modulation format independent. To a good extent, techniques based on DOP evaluation reduce hardware complexity [7].

Unfortunately, PMD effect is one of critical challenges in next-generation microwave fiber links after the successful mitigation of chromatic dispersion. Due to the PMD in fiber caused by DGD between two input PSPs traveling through the fiber, the power of microwave signal fades periodically at receiver, which leads to bit error ratio (BER) increasing [2,4]. It is necessary to achieve better transmission by compensating the PMD of optical fiber. Furthermore, due to environmental variations, such as temperature, vibration, and stress, both DGD and PMD drift randomly with time [6]. Recently, a technique using the DOP of received signal as the feedback signal has been demonstrated. It is promising in compensation the PMD of systems, since no any high-speed electrical circuits are required and independent to the bit rate [6,7]. Due to existence of PMD in optical communication link, the DOP of optical signal will degrade even if the input one is completely polarized, since each frequency component of input signal experiences different evolution of polarization state. Therefore, the DOP can be used as feedback control signal for PMD compensation [8].



## 2. Theoretical Modeling for Single Section

Consider a generic input field $\vec{A}_{in}(T) = A_{in}(T)|s>$ where $A_{in}(T)$ represents the field complex amplitude, and $|s>$ represents the input Jones polarization vector, which depends on the azimuth $\theta$ and the ellipticity $\eta$ of the input SOP. Mathematically, the SOP is defined as [9]

$$|s> = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\eta \pm i\sin\theta\sin\eta \\ \sin\theta\cos\eta \mp i\cos\theta\sin\eta \end{bmatrix} \quad (1)$$

The corresponding SOP vector in Stokes space will be [10]

$$\vec{S} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} |a|^2 + |b|^2 \\ |a|^2 - |b|^2 \\ ab^* + a^*b \\ i(ab^* - a^*b) \end{bmatrix} \quad (2)$$

Since different frequency components of the input broadband signal will undergo different polarizations evaluation when they propagate through the fiber, the signal DOP will degrade. Consequently, the weighted average of the Stokes vector is necessary, which calculated by multiplication the normalized spectral intensity function $|f(w)|^2 = |A_{in}(w)|^2$ and integrating over $w$. The DOP will be [5]

$$DOP = \frac{\sqrt{<s_1>^2 + <s_2>^2 + <s_3>^2}}{<s_0>} \qquad (3)$$

The input SOP does not depend on the frequency, such that the DOP of the input field will be *1*, but the output SOP form optical fiber with PMD is a function of frequency such that we are expected to be the DOP < *1*.

The matrix $T_{PMD}(w)$ is responsible for PMD effects, which is a $2\times 2$ unitary matrix that is defined as [10]

$$T_{PMD} = \exp[-\frac{iw}{2}\vec{\tau}(w).\vec{\sigma}] = \begin{bmatrix} u_1 & u_2 \\ -u_2^* & u_1^* \end{bmatrix} \qquad (4)$$

where $\vec{\tau}(w) = \tau\,\hat{r}$ is the PMD vector, $|\vec{\tau}(w)| = \tau$ is the DGD between the PSPs, $\hat{r}$ represents the direction of slow PSP, where the parameters $\hat{r}$ and $\tau$ are random, and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli spin matrices which can be defined as [7]

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$\sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad (5)$$

Here $\vec{\tau}\cdot\vec{\sigma} = \tau_1\sigma_1 + \tau_2\sigma_2 + \tau_3\sigma_3$ is a $2\times 2$ matrix. Eq.(4) may be reformed as

$$T_{PMD} = \begin{bmatrix} C - ir_1 S & -i(r_2 - ir_3) \\ -i(r_2 + ir_3) & C + ir_1 S \end{bmatrix} \qquad (6)$$

where $C$ and $S$ represent $\cos(w\tau/2)$ and $\sin(w\tau/2)$, respectively, and $r_i$ , $i = 1,2,3$ are the components of $\hat{r}$. Now, the output Jones vector after the PMD elements will be

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} C - ir_1 S & -i(r_2 - ir_3) \\ -i(r_2 + ir_3) & C + ir_1 S \end{bmatrix} \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \end{bmatrix} \qquad (7)$$

Typically, the normalized Gaussian pulse with initial width $T_0$ has the Fourier transform

$$f(w) = \sqrt{2\pi T_0^2}\exp(-w^2 T_o^2/2) \qquad (8)$$

The Stokes parameters of the output field are averaged in the whole signal frequency domain, then

$$<s_0> = \frac{1}{2\pi}\int_{-\infty}^{\infty}[|E_x|^2 + |E_y|^2]\,|f(w)|^2\,dw \qquad (9a)$$

$$<s_1> = \frac{1}{2\pi}\int_{-\infty}^{\infty}[|E_x|^2 - |E_y|^2]\,|f(w)|^2\,dw \qquad (9b)$$

$$<s_2> = \frac{1}{2\pi}\int_{-\infty}^{\infty}[E_x E_y^* + E_x^* E_y]\,|f(w)|^2\,dw \qquad (9c)$$

$$<s_3> = \frac{1}{2\pi}\int_{-\infty}^{\infty}i\,[E_x E_y^* - E_x^* E_y]\,|f(w)|^2\,dw \qquad (9d)$$

Now, it is very easy to prove that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}C^2\,|f(w)|^2\,dw = T_o\sqrt{\pi}\,\frac{1 + e^{-\tau^2/4T_0^2}}{2} \qquad (10a)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}S^2\,|f(w)|^2\,dw = T_o\sqrt{\pi}\,\frac{1 - e^{-\tau^2/4T_0^2}}{2} \qquad (10b)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}SC\,|f(w)|^2\,dw = 0 \qquad (10c)$$

Substituting Eqs.(7) and (8) into (9) and using (10), we obtain

$$< s_0 >= T_o\sqrt{\pi} \qquad (11a)$$

$$\begin{bmatrix} < s_1 > \\ < s_2 > \\ < s_3 > \end{bmatrix} = \frac{T_o\sqrt{\pi}}{2} Q \begin{bmatrix} 1+e^{-\tau^2/4T_0^2} \\ 1-e^{-\tau^2/4T_0^2} \\ 0 \end{bmatrix} \qquad (11b)$$

where

$$Q = \begin{bmatrix} s_1 & 2r_1\cos\psi - s_1 & 2(\hat{r}\times\hat{S})_1 \\ s_2 & 2r_2\cos\psi - s_2 & 2(\hat{r}\times\hat{S})_2 \\ s_3 & 2r_3\cos\psi - s_3 & 2(\hat{r}\times\hat{S})_3 \end{bmatrix}$$

Note that, $(\hat{r}\times\hat{S})_i$ , $i = 1,2,3$ are the components of the vector $\hat{r}\times\hat{S}$, and $\psi$ is the angle between $\vec{S}$ and $\hat{r}$. Eq.(11) may be rewritten in the reduced form

$$< s_i >=< s_0 > [s_i\gamma + r_i\cos\psi(1-\gamma)] \qquad (12)$$

where $\gamma = \exp(-\tau^2/4T_0^2)$

and $i=1,2,3$. The substitution of Eq.(12) into (3) will yield

$$DOP = \sqrt{\cos^2\psi + \gamma\sin^2\psi} \qquad (13)$$

It is important to note that the DOP is a function of an angle between the PSP vector and the input SOP vector. That is, if $\vec{S}$ is parallel to $\pm\hat{r}$, then the DOP will not be affected, while the orthogonality between $\vec{S}$ and $\pm\hat{r}$ will obtain the worst case $DOP = \sqrt{\gamma} < 1$. Note that, in the real system $\tau \le T_0/4$, such that the worst case is $DOP = e^{-1/32} = 0.9692$. Theoretically, $0.9692 \le DOP \le 1$ are expected depending on the initial pulse width, DGD, and the input SOP.

## 3. Generalization the Result

The above management may be generalized for any $N$ concatenation sections to obtain

$$DOP_j = \sqrt{\cos^2\psi_j + e^{-\tau_j^2/4T_{0j-1}^2}\sin^2\psi_j} \qquad (14)$$

where $j$ represents the section number and $\psi_j = \cos^{-1}(\hat{r}_j\cdot\hat{S}_{j-1})$. The vectors $\hat{r}_j$ and the DGD $\tau_j$, $j = 1,2,..,N$ are randomly generated at each section. The input SOP vector $\hat{S}_0$ are used to generate the next SOP using the recursive formula $\hat{S}_j = T_{PMDj}\hat{S}_{j-1}$. After each section, the width of output Gaussian pulse are determined in order to find its width, i.e. $T_{oj}$. This width may be found recessively using the relation [11]

$$T_{0j} = \sqrt{T_{0j-1}^2 + \left(\frac{\tau_j}{2}\sin\psi_j\right)^2} \qquad (15)$$

where $T_{00} = T_0$ is the original pulse width. In general,

$$DOP_j = \sqrt{\cos^2 \psi_j + \gamma_j \sin^2 \psi_j} \qquad (16)$$

where

$$\gamma_j = \exp\left( \frac{-\tau_j^2}{\left(4T_{0j-1}^2 + \tau_{j-1}^2 \sin^2 \psi_{j-1}\right)} \right)$$

and $\tau_0 = 0$ and $\theta_0 = 0$.

Eq.(16) is considered as the main conclusion of this paper. To our best knowledge, there are many published studies [12-15] related to the theoretical treatment of the signal DOP in presence of PMD, which are introduced many forms of the DOP, but all these forms may be considered as a partial form of Eq.(16) above.

## 4. Results and Discussion

The above managements are carried as follows to simulate the optical fiber with PMD using N concatenation model that may explain the behavior of DOP: 1) The random parameters $\tau_1$ and $\hat{r}_1$ are generated randomly. In turn, $\psi_1 = \cos^{-1}(\vec{S}_0 \cdot \hat{r}_1)$ will be computed and the next SOP $\vec{S}_1 = T_{PMD1}\vec{S}_0$ will be found, where $T_{PMD1}$ represents the average over the whole signal frequency domain, 2) The width of the resulted pulse $T_{01}$ and the resulted $DOP_1$ will be determined using Eqs.(15) and (16), respectively, 3) The processes at (1) and (2) are repeated for the second, third,.. etc sections, and 4) The above steps are repeated over many fibers to explain the accurate distribution of DOP.

Fig.(1) illustrates the random variation of DOP along the fiber for different values of PMD parameter. It is clear that the DOP will be 1 for $D_p = 0$, the increasing of $D_p$ will change randomly below the value 1. The amount of variation will be more for larger value of $D_p$. The reason for this behavior is attributed to the non coincidence between the SOP and the local PMD vectors. Fig.(2) illustrates the random variation of DOP along the fiber for two values of $T_0$ parameter. The behavior explains the fact that the smaller signal $T_0$ has the larger randomness. This issue may be illustrated with respect to the relation between the pulse width in time and frequency domains. Fig.(3) presents the DOP as a function of DGD for different values of pulse width. It is very clear that the DOP is a decreasing function of DGD, where the amounts of lowering are controlled by $T_0$. The smaller $T_0$ corresponds to the more affected DOP since the smaller $T_0$ means that the bandwidth of the frequency spectrum is the larger, hence the entire range of frequency is the larger. Figs.(5) and (6) illustrate the probability density function of the DOP using different values of $T_0$ and $D_p$. It is clear that the distribution will be shifted to left

ISSN 2229-5518

by increasing $D_p$ and vice versa. On the other hand, the increasing of $T_0$ will limit the distributions a smaller range since the final distribution is calculated over the entire range of frequency.

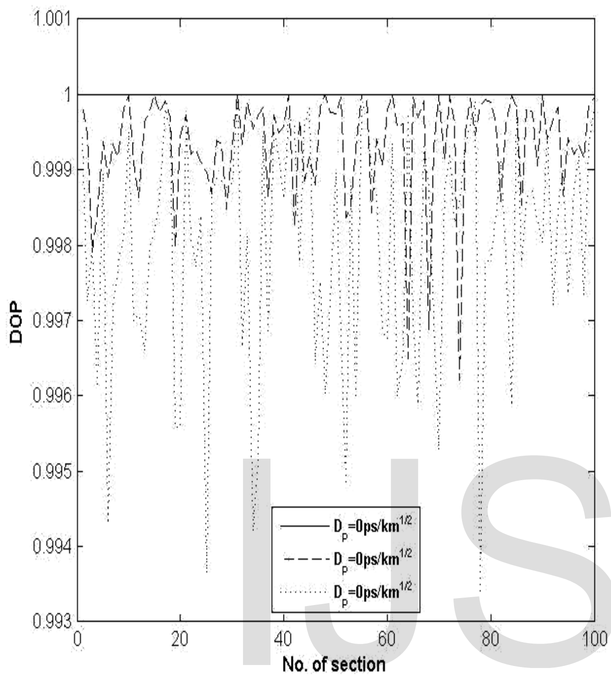


**Fig.(1)**: Evaluation of DOP with no. of section for different values of $D_p$ and $T_0 = 10\,ps$.

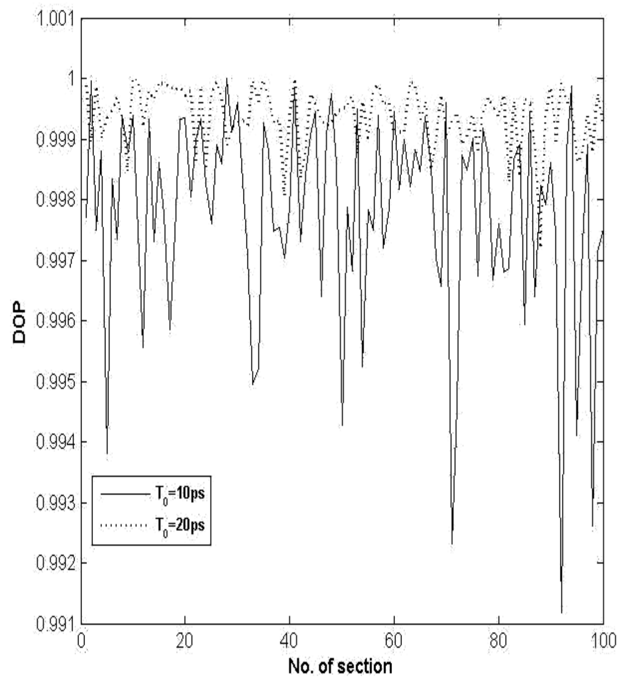


**Fig.(2)**: Evaluation of DOP with no. of section for different values of $T_0$ and $D_p = 1 ps/\sqrt{km}$ .

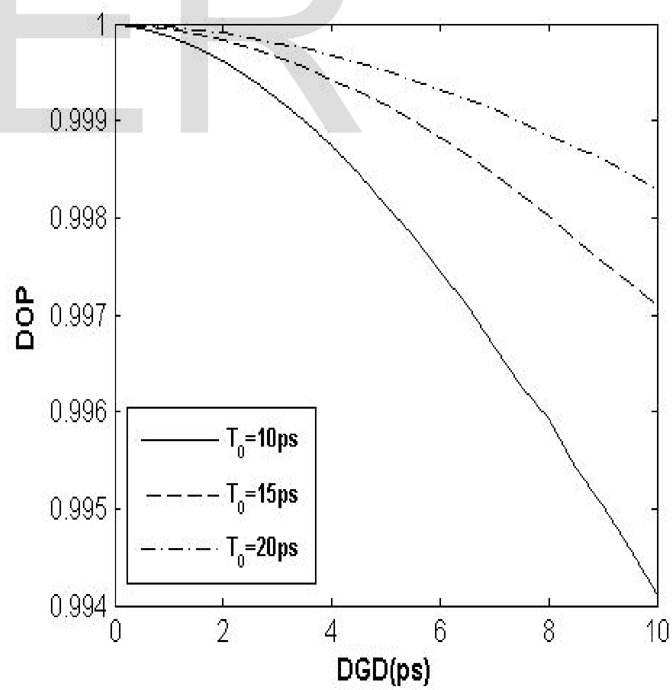


**Fig.(3)**: DOP as a function of DGD for different input pulse widths.

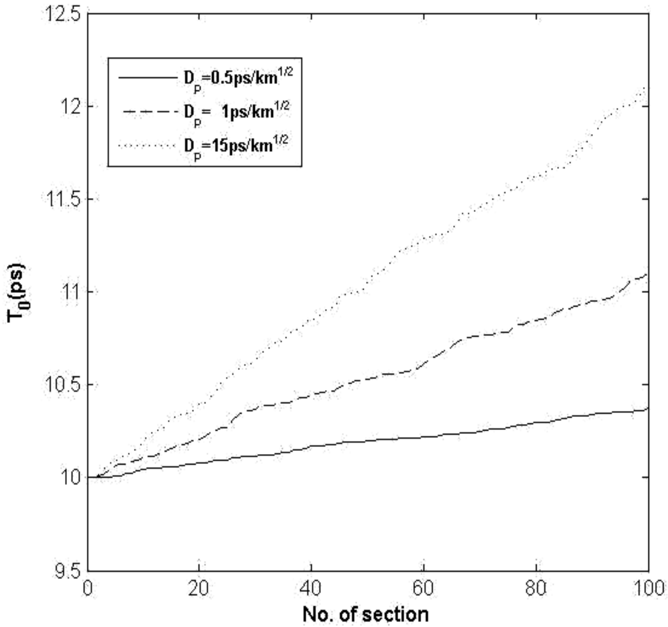


**Fig.(4)**: Pulse width with no. of section for different values of $T_0$ and $D_p = 1 ps/\sqrt{km}$ .

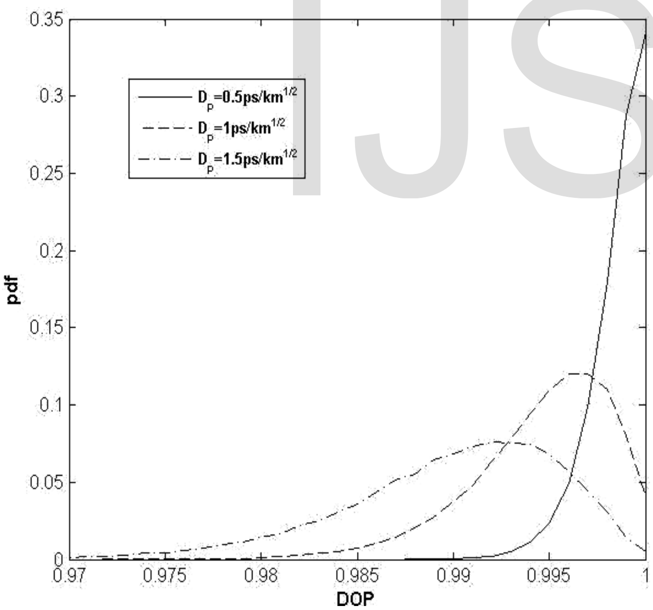


**Fig.(5)**: probability density function of DOP for different values of $D_p$ and $T_0 = 10 ps$ .

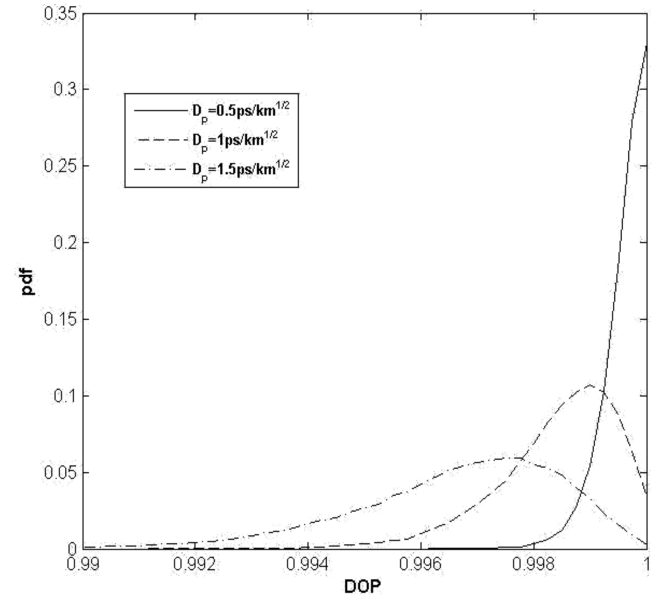


**Fig.(6)**: probability density function of DOP for different values of $D_p$ and $T_0 = 20 ps$ .



## 5. Conclusions

As a conclusion, the present model is a very good method to expect the resulted DOP depending on the local PMD vectors and the initial pulse width, where the increasing of $D_p / T_0$ will reduce/ raise the amount of variation of DOP and vice versa.

**References**

[1] J. Gordon and H.Kogelnik, "PMD fundamentals: polarization mode dispersion in opt ical fibers", Proc Nat l Acad Sci, 97(9), 4541-50, 2000.
[2] C. D. Poole and R. E. Wagner, “Phenomenological approach to polarizat ion dispersion in long single-mode fibers”, Electronics Letters, 22(19), 1029-1030, 1986.
[3] L. Chen L, Z. Zhang and X. Bao, "Combined PMD–PDL effects on BERs in simplified opt ical system: an analytical approach", Opt Express, 15(5), 2106-19, 2007.
[4] M. Shtaifand O. Rosenberg, "Polarization dependent loss as a waveform distort ion mechanism and its effect on fiber optic systems", J Lightwave Tech, 23(2), 923-30, 2005.
[5] G.P. Agrawal, “Lightwave Technology”, Wiley-Interscience, 2005.
[6] D. S. Waddy, L. Chen, X. Bao and D. Harris, “Statistics of relative orientation of principal states of

polarization in the presence of PMD and PDL", Proceedings of SPIE, 5260, 394-396, 2003.

[7] Hassan A. Yasser, "Conventional Polarization Effects on the Propagated pulses in Optical Fibers Systems", Al-Mustansiriya J. Sci., Vol. 20, No. 2, 2009

[8] Hassan Abid Yasser and Nizar Salim Shnan, "Theoretical Analysis of the Complex Polarization Mode Dispersion Vector in Single Mode Fibers", American Journal of Modeling and Optimization, Vol. 1, No. 2, 2013.

[9] Nazar S. Shnan, "Performance of Single Mode Fiber System in Presence the Conventional Polarization Effects", Ph. D. Thesis, University of Al-Mustansiriya, Education College, 2012.

[10] Hassan A. Yasser, "The dynamics of state of polarization in the presence of conventional polarization effects in optical fibers", Optics & Laser Technology

[11] Hassan A. Yasser, "Polarization Losses in Optical Fibers", in Recent progress in optical fiber research, M. Yasin, S. Harun, and H. Arof, Eds. Croatia: Intech, pp.103-126, **2012**. , 2010.

[12] H. Liu, X. Zhang, and K. Chen, "Degree of Polarization technique in PMD compensation of optical microwave transmission systems", Optics Communications, 236, 109-114, 2004.

[13] D. Derickson, "Fiber Optic Test and Measurements", Prentice Hall, 1998.

[14] A. Orlandini and L. Vincetti, "A Simple and Useful Model for Jones Matrix to Evaluate Higher Order polarization Mode Dispersion effects", IEEE Phot. Tech. Let., Vol.13, No.9, 2001.

[15] E. Forestieri and L. Vincetti, "Exact Evaluation of the Jones Matrix of a Fiber in the Presence of PMD of any Order", IEEE Phot. Tech. Let., Vol.19, No.12, 2001..